Available online at www.sciencedirect.com
Acta Mathematica Scientia 2009,**29B**(5):1241–1250
http://actams.wipm.ac.cn

POINTWISE CONVERGENCE FOR EXPANSIONS IN SPHERICAL MONOGENICS*

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Abstract We offer a new approach to deal with the pointwise convergence of Fourier-Laplace series on the unit sphere of even-dimensional Euclidean spaces. By using spherical monogenics defined through the generalized Cauchy-Riemann operator, we obtain the spherical monogenic expansions of square integrable functions on the unit sphere. Based on the generalization of Fueter's theorem inducing monogenic functions from holomorphic functions in the complex plane and the classical Carleson's theorem, a pointwise convergence theorem on the new expansion is proved. The result is a generalization of Carleson's theorem to the higher dimensional Euclidean spaces. The approach is simpler than those by using special functions, which may have the advantage to induce the singular integral approach for pointwise convergence problems on the spheres.

Key words spherical monogenics; pointwise convergence of Fourier-Laplace series; generalized Cauchy-Riemann operator; unit sphere; generalization of Fueter's theorem

2000 MR Subject Classification 42B05; 30G35

1 Introduction

$f \in L^1(-\pi, \pi$
c_k

$$c_k \quad \frac{}{\pi} \int_{-\pi}^{\pi} f(t \quad {}^{-\mathrm{i}kt} \quad t, \qquad k \in \mathbf{Z}, \qquad ($$

$\mathbf{Z}$ $s_N(f \ (x$

$$s_N(f \ (x \quad \sum_{|k| \le N} c_k \quad {}^{\mathrm{i}kx}, \qquad x \in \ -\pi, \pi \ , \ N \in \mathbf{N}_0, \qquad ($$

*Received August 23, 2007. Sponsored by Research Grant of the University of Macau No. RG024/03-04S/QT/FST

f $\mathbf{N}_0$ **T**

$$s_N(f \;(x \quad \frac{}{\pi}\int_{-\pi}^{\pi} f(t \; D_N(x-t \quad t, \qquad ($$

$$D_N(x \quad \begin{cases} \dfrac{(N \quad \frac{1}{2} \; x}{\frac{x}{2}} & x \in \; -\pi, \pi \setminus \{ \; \}, \\ N \quad - & x \end{cases}$$

N $L^2(-\pi, \pi \quad \subset L^1(-\pi, \pi$ L^2

T C

T **1.1** $f \in L^2(-\pi, \pi$

$$s_N(f \;(x \; \to f(x \;, \quad . \; . \; x \in \; -\pi, \pi \;, \quad N \to \quad \infty.$$

C **T** $f \in L^p(-\pi, \pi \quad < p < \infty$ S^n $(n$ $\mathbf{R}_1^n$

$$\mathbf{R}_1^n \quad \{x \quad x_0 \quad \underline{x} \mid x_0 \in \mathbf{R}, \; \underline{x} \in \mathbf{R}^n\}$$

f S^n $f \in L^2(S^n$

$$f \sim \sum_{k=0}^{\infty} f_k, \qquad ($$

f_k k **T**

(

T n ((

n

(n

f^0 $\overline{f^0(z}$ $f^0(z$

(f^0 u v

u v $v(x_0,$

f^0

τ

$$\tau(f^0 \quad \kappa^{-1}\Delta^{\frac{n-1}{2}}\vec{f^0},$$

Δ DD D $\frac{\partial}{\partial x_0}$ $_0-\frac{\partial}{\partial x_1}$ $_1-\cdots-\frac{\partial}{\partial x_n}$ $_n$ κ_n ($^{n-1}$ $^2(\frac{n+1}{2}$

$\tau((\cdot\ ^{-1}$ E (

T $\Delta^{\frac{n-1}{2}}$

M $\mathcal{S}'\to\mathcal{S}'$ $m(\xi$ (π $|\xi|$ $^{n-1}$

$$Mf \quad \mathcal{R}(m\mathcal{F}f\ ,$$

$$\mathcal{F}f(\xi \quad \int_{\mathbf{R}_1^n} {}^{2\pi \mathrm{i}\langle x,\xi\rangle} f(x \quad x,$$

$$\mathcal{R}h(x \quad \int_{\mathbf{R}_1^n} {}^{-2\pi \mathrm{i}\langle \xi,x\rangle} h(\xi \quad \xi.$$

$\mathcal{F}$ $\mathcal{R}$

n $\Delta^{\frac{n-1}{2}}$

$\mathbf{R}_1^n$ $n\in\mathbf{Z}^+$ (**T**

n (

P **2.1** $f^0(z$ $u(s,t$ $v(s,t$

$O\subset\mathbf{C}$. **T** $\tau(f^0$ $\vec{O}$.

f^0 $z^k, k\in\mathbf{Z}$

$$P^{(-k)} \quad \tau((\cdot\ ^{-k}\ , \quad P^{(k-1)} \quad I(P^{(-k)}\ , \quad k\in\mathbf{Z}^+,$$

$\mathbf{Z}^+$ I

$$I(f\ (x \quad E(x\ f(x^{-1}\ .$$

n (

$$\tau((\cdot\ ^{(k)} \quad P^{(k)},\ k \quad -\ ,-\ ,\cdots \qquad ($$

$$\tau((\cdot\ ^{(k+n-1)} \quad P^{(k)},\ k \quad ,\ ,\ ,\cdots. \qquad ($$

k

$$P^{(k)}(y^{-1}x\ E(y$$

x y ($E(y\ n(y$ C

$$\int_{S^n} P^{(k)}(y^{-1}x \quad \sigma(y \quad , \quad |x| \quad ,\ k\ / \qquad ($$

$P^{(0)}$ $I(P^{(-1)}$ $I(E$,

$$\frac{}{\omega_n}\int_{S^n} P^{(0)}(y^{-1}x \quad \sigma(y \quad , \quad |x| \quad . \qquad ($$

3 Expansions of the Fourier-Laplace Series and Dirichlet Kernels

C

(k f_k,

$f \in L^2(S^n$ (

f_k g_k h_k, (

g_k k h_k

$-k$ $-n$ (

$$g_k(x \quad \frac{}{\omega_n}\int_{S^n} P^{(k)}(y^{-1}x\ f(y \quad \sigma(y\ ,$$

$$h_k(x \quad \frac{}{\omega_n}\int_{S^n} P^{(-k)}(y^{-1}x\ f(y \quad \sigma(y\ .$$

$P^{(k)}(y^{-1}x$ $P^{(k)}$

(**T** ($S_N f(x$

$$S_N f(x \quad \frac{}{\omega_n}\int_{S^n} D_N^{(n+1)}(y^{-1}x\ f(y \quad \sigma(y\ , \qquad ($$

$$D_N^{(n+1)}(x \quad \sum_{|k|\le N} P^{(k)}(x \qquad ($$

N $\mathbf{R}_1^n$. n ((

$$D_N^{(n+1)}(x \quad \tau\Big(\sum_{k=-N}^{N+n-1}(\cdot\ ^{(k)}\Big)(x\ . \qquad ($$

n $(n-\quad /$

T $D_N^{(n+1)}(x$

P **3.1** $h(t,s$ t s O $s>\ ,t>\ .$

$n \in \mathbf{Z}^+$ x x_0 $\underline{x} \in \mathbf{R}_1^n$.

$$H(x \quad \Delta^{(n-1)/2}h(x_0,|\underline{x}|\ ,$$

Δ n

$$H(x \quad (n- \quad (\frac{}{s}\partial_s\ ^{(n-1)/2}h(t,s\ |_{t=x_0,s=|\underline{x}|}, \qquad ($$

H $x_0,x_1,\cdots,x_n$ $\mathbf{R}_1^n$.

(

C ^{n}

$f \in L^2(S^n$ f (

(g_k h_k

$$f_k(x \quad \frac{}{\omega_n} \int_{S^n} (P^{(k)} \quad P^{(-k)} \ (y^{-1}x \ f(y \quad \sigma(y \ .$$

f $P^{(k)}$ $P^{(-k)}$ $f_k,$

$P^{(k)}$ $P^{(-k)}$ T

$D_N^{(n+1)}$ J T T ((

4 Main Results

$\mathcal{D}_N^{(n+1)}(x$ S^n

θ θ x_0 $\theta \in$ $,\pi$ $\mathcal{D}_N^{(n+1)}(y^{-1}x$ S^n

$(y^{-1}x$ $x, y \in S^n$ $(y^{-1}x$ $(yx$

$\langle y, x\rangle$ x S^n y x θ y θ y

x θ $(y, x$ y^{-1} y x θ y θ $(y^{-1}x$ $\langle y, x\rangle$ θ

$f \in L^2(S^n$ $(n -$ y

$(y, x$ θ **T**

$$\Phi_x(f\ (\theta \quad \frac{}{\omega_{n-1}} \int_{S^{n-1}} f(x \quad \theta \quad y \quad \theta \quad \sigma_{n-1}(y\ ,$$

y S^{n-1} $\sigma_{n-1}(y$

S^{n-1} $\Phi_x(f\ (\theta$ f x θ

f

$\Phi_x(f\ (\theta$ $\Phi_x(\theta$ $\mathcal{W}_2^{l,1}($ $,\pi$

$$\mathcal{W}_2^{l,1}(\ \ ,\pi \qquad \Big\{g \in L^2(\ \ ,\pi\ \ |(\frac{\partial}{\partial\theta}\ ^k g \in L^1(\ \ ,\pi\ \ ,\ k \quad ,\ ,\cdots, l\Big\}.$$

$\Phi_x($ $_{\theta\to 0}\Phi_x(\theta$ **T**

T **4.1** x S^n $f \in L^2(S^n$ $\Phi_x($

$\Phi_x(\theta\ \in \mathcal{W}_2^{l,1}(\ \ ,\pi$ $_{N\to\infty} S_N(f\ (x \quad \Phi_x($. f

x $_{N\to\infty} S_N(f\ (x \quad f(x$.

P $_{N\to\infty}(S_N(f\ (x\ - \Phi_x($. (

(

$$\frac{}{\omega_n}\int_{S^n} D_N^{(n+1)}(y^{-1}x \quad \sigma(y \qquad .$$

$$_{N\to\infty} \frac{}{\omega_n}\int_{S^n} D_N^{(n+1)}(y^{-1}x\ (f(y\ - \Phi_x(\quad \sigma(y \qquad .$$

T N

$\mathcal{D}_N^{(n+1)}$ $\mathcal{D}_N^{(2l+2)}$

$$\frac{}{\omega_n}\int_{S^n} \mathcal{D}_N^{(2l+2)}(y^{-1}x\ (f(y\ - \Phi_x(\quad \sigma(y\ .$$

$\mathcal{D}_N^{(2l+2)}(y^{-1}x$

$$\frac{}{\omega_n}\int_0^{\pi}(\quad \theta\ ^{(2l+2-2)}\Big[\kappa_n^{-1}(\ \ l \quad \sum_{j=1}^{l}\sum_{i=1}^{j} C_l^j(\quad \theta\ ^{j-2l} Q_i^{(j)}(\quad \theta, \quad \theta\ (\frac{\partial}{\partial\theta}\ ^i U_N\Big]$$
$$\cdot(\Phi_x(\theta\ - \Phi_x(\quad \theta$$

$$\frac{}{\omega_n}\int_0^{\pi}(\quad \theta^{\,(2l+2-2)}\Big[\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}\sum_{i=1}^{j}C_l^j(\quad \theta^{\,j-2l}R_i^{(j)}(\quad \theta, \quad \theta\ (\frac{\partial}{\partial\theta}\ ^{i}V_N\Big]$$

$$\cdot(\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \theta$$

$$\frac{}{\omega_n}\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}\sum_{i=1}^{j}C_l^j\int_0^{\pi}(\frac{\partial}{\partial\theta}\ ^{i}U_N\ (\quad \theta^{\,j}Q_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \theta$$

$$\frac{}{\omega_n}\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}\sum_{i=1}^{j}C_l^j\int_0^{\pi}(\frac{\partial}{\partial\theta}\ ^{i}V_N\ (\quad \theta^{\,j}R_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \theta.$$

T

$$\lim_{N\to\infty}(S_N(f\ (x\ -\mathbf{\Phi}_x($$

$$\lim_{N\to\infty}\frac{}{\omega_n}\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}\sum_{i=1}^{j}(- \quad ^{i}C_l^j\int_0^{\pi}U_N(\quad \theta, \quad \theta\ (\frac{\partial}{\partial\theta}\ ^{i}$$

$$\cdot\Big[(\quad \theta^{\,j}Q_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \Big]\ \theta$$

$$\lim_{N\to\infty}\frac{}{\omega_n}\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}\sum_{i=1}^{j}(- \quad ^{i}C_l^j\int_0^{\pi}V_N(\quad \theta, \quad \theta\ (\frac{\partial}{\partial\theta}\ ^{i}$$

$$\cdot\Big[(\quad \theta^{\,j}R_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \Big]\ \theta$$

$$\lim_{N\to\infty}I_1 \quad \lim_{N\to\infty}I_2.$$

$\mathbf{\Phi}_x(\theta\ \in\mathcal{W}_2^{l,1}(\ ,\pi \quad \mathbf{\Phi}_x(\theta\ -$ $\mathbf{\Phi}_x($ $\mathcal{W}_2^{l,1}(\ ,\pi$ $(\frac{\partial}{\partial\theta}\ ^{i}(\quad \theta^{\,j}R_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x($ $\le i\le j$ V_N $\frac{\sin(N+\frac{n}{2})\theta\sin\frac{n-1}{2}\theta}{\sin\frac{\theta}{2}}$ $\lim_{N\to\infty}I_2$.

I_1 U_N $\frac{\sin(N+\frac{n}{2})\theta\cos\frac{n-1}{2}\theta}{\sin\frac{\theta}{2}}$ θ j $\le j\le l$ $\sum_{i=1}^{j}$ **T** $(\quad \theta^{\,j}$ $\frac{\partial}{\partial\theta}$ j $\le i\le j-$ $(\frac{\partial}{\partial\theta}\ ^{i}$ $(\quad \theta^{\,j}Q_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x($ $(\quad \theta^{\,m_1}$ $m_1\ge$ i j $(\frac{\partial}{\partial\theta}\ ^{j}$ $(\quad \theta^{\,j}Q_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x($ $(\quad \theta^{\,m_2}$ $m_2\ge$ $j\ (\quad \theta^{\,j}Q_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad .$ I_1 $\sum_{i=1}^{j}$ I_1

$$\lim_{N\to\infty}I_1 \quad \lim_{N\to\infty}\frac{}{\omega_n}\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}(- \quad ^{j}C_l^j\int_0^{\pi}U_N(\quad \theta, \quad \theta$$

$$\cdot\ j\ (\quad \theta^{\,j}Q_j^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \theta$$

$$\lim_{N\to\infty}\frac{}{\omega_n}\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}(- \quad ^{j}j\ C_l^j\int_0^{\pi}\frac{(N \quad \frac{n}{2}\ \theta \quad \frac{n-1}{2}\theta}{\frac{\theta}{2}}$$

$$\cdot(\quad \theta\ {}^{j}Q_j^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \theta$$

$$\frac{\pi}{\omega_n}\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}(- \quad {}^{j}j\ C_l^j \ {}_{N\to\infty}\ \frac{}{\pi}\int_0^{\pi} \frac{((N \quad \frac{n-1}{2} \quad \frac{1}{2}\ \theta}{\frac{\theta}{2}}\ {}_x^{(j)}(\theta \quad \theta,$$

${}_x^{(j)}(\theta$ $\frac{n-1}{2}\theta(\quad \theta\ {}^{j}Q_j^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x($ $\leq j \leq l$ $\mathbf{\Phi}_x(\theta \in$ $L^2(\ ,\pi$ ${}_x^{(j)}(\theta \in L^2(\ ,\pi$ $\leq j \leq l$ C

$$_{N\to\infty} I_1 \quad \frac{\pi}{\omega_n}\kappa_n^{-1}(\ l \quad \sum_{j=1}^{l}(- \quad {}^{j}j\ C_l^j\ {}_x^{(j)}(\quad .$$

T $_{N\to\infty} S_N(f\ (x \quad \mathbf{\Phi}_x(\ .$

(L^p $< p < \infty$

T

E a 4.1 n l **T** $\mathbf{\Phi}_x(\theta$ $(-\theta,\theta$

$$\mathbf{\Phi}_x(\theta \quad - f(\theta_x \quad \theta \quad f(\theta_x-\theta\ ,$$

θ_x x e_1 $\mathcal{D}_N^{(2)}$ J D_N **T** $\mathbf{\Phi}_x \in L^2(\ ,\pi$ $\frac{1}{2}\ f(\theta_x \quad \theta$ $f(\theta_x-\theta \in L^2(\ ,\pi$ x **T** C

E a 4.2 l n **T** q S^3 $f \in L^2(S^3$ $\mathbf{\Phi}_q($ $\mathbf{\Phi}_q(\theta \in L^2(\ ,\pi$ ${}_q(\theta$ ${}_q(\theta$ $(\ ,\pi$ $_{N\to\infty} S_N(f\ (q \quad \mathbf{\Phi}_q($ $_{N\to\infty} S_N(f\ (q \quad f(q \quad f$ q **T** $\mathbf{\Phi}_q(\theta$ $,\pi$ $\mathbf{\Phi}_q(\theta$ $,\pi$ $\mathbf{\Phi}_q(\theta \in L^2(\ ,\pi$ $,\pi$ $l \geq$ **T**

$$\begin{cases} (\quad \frac{n-1}{2}\theta(\quad \theta\ {}^{j}Q_j^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \in L^2(\ ,\pi\ , \quad \leq j \leq l \\ (\quad (\frac{\partial}{\partial\theta}\ {}^{i}\ (\quad \theta\ {}^{j}Q_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \in L^1(\ ,\pi\ , \quad \leq i \leq j \leq l \\ (\quad (\frac{\partial}{\partial\theta}\ {}^{i}\ (\quad \theta\ {}^{j}R_i^{(j)}(\quad \theta, \quad \theta\ (\mathbf{\Phi}_x(\theta\ -\mathbf{\Phi}_x(\quad \in L^1(\ ,\pi\ , \quad \leq i \leq j \leq l. \end{cases}$$

$\mathbf{\Phi}_x(\theta \in \mathcal{W}_2^{l,1}(\ ,\pi$ **T**

C a 4.1 f l S^n $l \geq$ $f \in C^l(S^n$

$$_{N\to\infty} S_N(f\ (x \quad f(x\ , \quad \forall x \in S^n.$$

P ($\mathbf{\Phi}_x(\theta \in$ $C^l((\ ,\pi$ $(\quad \theta\ {}^{j}(\frac{\partial}{\partial\theta}\ {}^{i}\mathbf{\Phi}_x(\theta \quad \leq i \leq j \leq l$ π

$\Phi_x(\theta$, π **T** $\Phi_x(\theta \in L^2($, π $(\frac{\partial}{\partial\theta})^i ($ $\theta)^j \Phi_x(\theta$
$\leq i \leq j \leq l$, π **T** (((
$Q_i^{(j)}$ $R_i^{(j)}$ θ θ f S^n

$$\lim_{N\to\infty} S_N(f)(x) = f(x), \quad \forall x \in S^n.$$

R c

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