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Advances in Applied Clifford Algebras

Half Dirichlet Problems and Decompositions of Poisson Kernels

We ad e a f C f d a eb a f [5]. I a , a e e a e e $a \in \mathbf{C}_{m+1}$ a e f

$$a = \sum_A a_A \mathbf{e}_A,$$

e e $a_A \in \mathbf{C}$, $A =< j_1, \cdots, j_l, 0 \leq j_1 < \cdots < j_l \leq m$, a d e e $\mathbf{e}_A = \mathbf{e}_{j_1} \cdots \mathbf{e}_{j_l}$ a e e *reduced products* f ba e e . F e e $|a| = \left(\sum_A |a_A|^2\right)^{1/2}$ e $a \in \mathbf{C}_{m+1}$. T e c a e $\overline{a}$ defi ed be e e d c f e c a $\mathbf{R}_{0,m+1}$ a d e c c a $\mathbf{C}$. Of ce a a ce e f da e a f e D ac e a (ee be) $\mathbf{R}^{m+1}$ de ed b $E(x)$; a e e e

$$E(x) = \frac{1}{A_{m+1}} \frac{\overline{x}}{|x|^{m+1}},$$

e e A_{m+1} e a ea f e m-d e a e e $\mathbf{R}^{m+1}$.

We d ce e f c

$$\alpha(x) = \frac{1}{2}\left(1 + i\frac{\partial\Phi(x)}{|\partial\Phi(x)|}\right), \quad \beta(x) = \frac{1}{2}\left(1 - i\frac{\partial\Phi(x)}{|\partial\Phi(x)|}\right),$$

e e ∂ e a D ac e a $\partial = \frac{\partial}{\partial_0}\mathbf{e}_0 + \frac{\partial}{\partial_1}\mathbf{e}_1 + \cdots + \frac{\partial}{\partial_m}\mathbf{e}_m$, a d i e a a a e c be e . T e ec $\partial\Phi(x)/|\partial\Phi(x)|$ a a ec f e face Σ e x Σ, de ed b

$$n = \frac{\partial\Phi(x)}{|\partial\Phi(x)|}.$$

We a e a e face e ab a d d de e ace e e f a a e -c ec ed, de ed b Ω. T n e e defi ed " a d" " a d" a f Ω. I e $\overline{\Omega}$ a e D b a d a D b be d ed.

F eac fi ed $x \in \Sigma$, $\alpha(x)$ a d $\beta(x)$ a e e a a e de e $\mathbf{C}_{m+1}$, .e.

$$\alpha^2(x) = \alpha(x), \quad \beta^2(x) = \beta(x);$$

$$\alpha(x)\beta(x) = \beta(x)\alpha(x) = 0;$$

$$\overline{\alpha}(x) = \alpha(x), \quad \overline{\beta}(x) = \beta(x).$$

M e e ,

$$\alpha(x) + \beta(x) = 1.$$

T e f c α a d β e e Ha d - ace ec a F e ea , ac efe e c d a f e f c . T e e e e- a f e ec e aced a a e e a . T e d a be f d [4], [9], [10], a d e [8]. I e, e e , e f e f c ade d ec e aced a 3140 e f c .

H e c f c f de ee λ, $f \in L^p(\Sigma), 1 < p < \infty$, fi d $W(x)$ c a

$$\begin{cases} \partial W(x) = 0 & x \in \Omega \\ \alpha(x)W(x) = \alpha(x)f(x) & x \in \Sigma, \end{cases} \tag{1.1}$$

$$\begin{cases} \partial W(x) = 0 & x \in \Omega \\ \beta(x)W(x) = \beta(x)f(x) & x \in \Sigma. \end{cases} \tag{1.2}$$

T e ca e $p = 1$ a d $p = \infty$ e e e de ca e a a a be ed a e . T e f ec be de ed e a D b e ba a d e e - a ace $\mathbf{R}^{m+1}$, e ec e . We a d c e D b a d e c e d dec f e P e e e e c e . I e d ec e a ba a d a ace a e e ca e f a D b a e f c e.

2. Half Dirichlet Problems in the Unit Ball

De e e e ba ce e ed a e b $B(1)$ e c e $\overline{B}(1)$. A b da f $B(1)$, e e e de ed b S_m. T e e e c f e e face $\Phi(x) = 1$, e e $\Phi(x) = |x|^2$. C de e e face $\Phi(x) = r \leq 1$ e c ed ba $\overline{B}(1)$. T e de e f c α a d β e e face a e

$$\alpha(x) = \frac{1}{2}(1 + ix), \quad \beta(x) = \frac{1}{2}(1 - ix).$$

We a e

$$\alpha(x)\beta(x) = \beta(x)\alpha(x) = 1 - |x|^2.$$

If, a , $x = r\omega$ $r = 1$, e. x e e e, e e a e

$$\alpha(\omega)\beta(\omega) = \beta(\omega)\alpha(\omega) = 0.$$

T e Ca c a f f a e b da da a f e b

$$C(f)(x) = \int_{S_m} \overline{C}(\omega)f(\omega)ds(\omega),$$

e e

$$C(x, \omega) = \frac{1}{A_{m+1}}\omega\frac{x - \omega}{|x - \omega|^{m+1}}$$

e Ca c e e e e.

T e a e e ad e e d c a f e ab e e a .

$$C(f) =< C , f_\Sigma,$$

e e a e defi e

$$< g, f_\Sigma = \int_\Sigma \overline{g}(\omega)f(\omega)ds(\omega).$$

T e e a D c b e α, f a ce, e c de e Ca c e a $C(f) = C_{S_m}(f)$ f e f c $2\alpha(\omega)f(\omega)$ de e ba e e $f(\omega)$ e b da da a e (1.1), e.

$$C(2\alpha f)(x) = \frac{1}{A_{m+1}}\int_{S_m}\frac{x-\omega}{|x-\omega|^{m+1}}\omega[2\alpha(\omega)f(\omega)]ds(\omega)$$
$$= \frac{1}{A_{m+1}}\int_{S_m}\frac{x-\omega}{|x-\omega|^{m+1}}\omega(1+i\omega)f(\omega)ds(\omega).$$

Se

$$W^\alpha(x) = C(2\alpha f)(x).$$

W e $x = r\xi$, e a e

( ) $W^\alpha(x)$ (f -) e $B(1)$;

( )

$$\lim_{r\to 1-} W^\alpha(r\xi) = W(\xi) \quad \text{(a defi)}$$
$$= \frac{1}{2}\left[2\alpha(\xi)f(\xi) + \mathcal{H}(2\alpha f)(\xi)\right],$$

e $\mathcal{H}$ e *Hilbert transformation* $C^\lambda(S_m)$ a d $L^p(S_m)$. ( ) e ca d P e S f . T e H e a f f a e e a f c f e e e defi ed be e c a a e e a

$$\mathcal{H}(f)(x) = p.v.\frac{2}{A_{m+1}}\int_{S_m}\frac{\xi-\omega}{|\xi-\omega|^{m+1}}\omega f(\omega)ds(\omega).$$

T e fac a $\mathcal{H}$ a $C^\lambda(S_m)$ $C^\lambda(S_m)$ a a b ded e a aced bac [11]; a d a $\mathcal{H}$ a $L^p(S_m)$ $L^p(S_m)$, ba ed, f $p = 2$, e P c e e e e e e; f $p \neq 2$ e efe [2] [4]. T e a f e P e S f f f c L^p a c e e ce f e b ded e f $\mathcal{H}$ e L^p ace (ee [12] [14]).

N e a e e f e e H e a f a f a a a . S e a ca e ab e defi ed $\mathcal{H}$ e *Cauchy singular integral* e e e. T e ead e e e H e a f a f e a a a e ea a e a a a f e b da a e f a (f -) e f c Ω (ee, f a ce, [1]). O e e - a ace e c ce c c de b d e a e f e e a d a c d ba .

N c de e f c $\alpha(x)W^\alpha(x)$. Ta e e b da , e a e

$$\lim_{r\to 1-} \alpha(x)W^\alpha(x) = \alpha(\xi)W^\alpha(\xi)$$
$$= \alpha^2(\xi)f(\xi) + \alpha(\xi)\mathcal{H}(\alpha f)(\xi)$$
$$= \alpha(\xi)f(\xi) + \alpha(\xi)\mathcal{H}(\alpha f)(\xi).$$

B , a

$$(1+i\xi)(\xi-\omega)\omega(1+i\omega) = 0, \tag{2.1}$$

e b a

$$\alpha(\xi)\mathcal{H}(\alpha f)(\xi) = 0.$$

C e e ,

$$\lim_{r\to 1-} \alpha(x)W^\alpha(x) = \alpha(\xi)f(\xi)$$

T e ef e, W^α e e b (1.1). S ,

$$W^\beta(x) = C(2\beta f)(x)$$

e e b (1.2).

T e ab e W^α a d W^β e b (1.1) a d (1.2), e ec e , e e e f e c a D b : G e b da da a $f \in C^\lambda(\Sigma), 0 < \lambda < 1$ $f \in L^p(\Sigma), 1 < p < \infty$, fi d $U(x)$ c a

$$\begin{cases} \Delta U(x) = 0 & x \in B(1) \\ U|_{S_m}(x) = f(x) & x \in S_m, \end{cases} \tag{2.2}$$

We eca e f fac .

( ) W^α a d W^β a e f - e $B(1)$; a d

( ) F a (f -) e f c f e e e $\Omega \subset \mathbf{R}^{m+1}$, e f c $xf(x)$ a Ω (ee, f a ce, [5]).

We e ef e a e a $\alpha(x)W^\alpha(x)$ a d $\beta(x)W^\beta(x)$ b a e a $B(1)$. He ce

$$U(x) = \alpha(x)W^\alpha(x) + \beta(x)W^\beta(x) \tag{2.3}$$

a $B(1)$. M e e ,

$$\begin{aligned} \lim_{r\to 1-} U(r\xi) &= \alpha(\xi)W^\alpha(\xi) + \beta(\xi)W^\beta(\xi) \\ &= \alpha(\xi)f(\xi) + \beta(\xi)f(\xi) \\ &= f(\xi). \end{aligned}$$

C e e , $U(x)$ e e D b (2.2).

T e W^α a d W^β e b (1.1) a d (1.2) a e e a dec f e P e e e e. N e a e f (2.2) e b

$$U(x) = \int_{S_m} P(x,\omega)f(\omega)ds(\omega),$$

e e

$$P(x,\omega) = \frac{1}{A_{m+1}} \frac{1-|x|^2}{|x-\omega|^{m+1}}, \quad x \in B(1),\ \xi \in S_m,$$

e P e e e e. T e W^α a d W^β a e defi e e f c

$$C^\alpha(\omega) = \frac{2}{A_{m+1}}\alpha(x)\frac{x-\omega}{|x-\omega|^{m+1}}\omega\alpha(\omega)$$

a d

$$C^\beta(\omega) = \frac{2}{A_{m+1}}\beta(x)\frac{x-\omega}{|x-\omega|^{m+1}}\omega\beta(\omega).$$

D e e e

$$(1+ix)(x-\omega)\omega(1+i\omega)+(1-ix)(x-\omega)\omega(1-i\omega)=2(1-|x|^2),$$

e b a e dec

$$P(x,\omega)=C^{\alpha}(\omega)+C^{\beta}(\omega), \tag{2.4}$$

a d e ce e $\alpha(x)W^{\alpha}(x)$ a d $\beta(x)W^{\beta}(x)$ a e e , e ec e , b

$$\alpha(x)W^{\alpha}(x)=\int_{S_m} C^{\alpha}(\omega)f(\omega)ds(\omega) \tag{2.5}$$

a d

$$\beta(x)W^{\beta}(x)=\int_{S_m} C^{\beta}(\omega)f(\omega)ds(\omega). \tag{2.6}$$

Remarks

( ) T e f (1.1) a d (1.2) f e ba ca e a e a ead d c ed e a e [7].

( ) F (2.3), e e (2.5) a d (2.6), a be e a

$$U(x)=\alpha(x)\int_{S_m}\overline{C}(\omega)(2\alpha f)(\omega)ds(\omega)+\beta(x)\int_{S_m}\overline{C}(\omega)(2\beta f)(\omega)ds(\omega),$$

d ca e fac a e c ca D b (2.2) f e ba a be ed b e Ca c a f a .

( ) I ba ed e (2.4) a e b a e dec (2.3). I deed, a a f e dec (2.4) a e e e a a ead be f d [7]. T e e a e e e

$$P(x,\omega)=P^{\alpha}(x,\omega)+P^{\beta}(x,\omega),$$

e e

$$P^{\alpha}(x,\omega)=\alpha(x)P(x,\omega), \quad P^{\beta}=\beta(x)P(x,\omega).$$

T e b e a a e D b f Δ $B(1)$ ca be ed b e Ca c a f a a e ade, a e e ed [7] T e e 3.2 ( ).

( ) I [6], ed a e e e b (2.2) ead

$$U(x)=F_1(x)+xF_2(x), \tag{2.7}$$

e e

$$F_1(x)=<S\ (\omega),f(\omega)_{S_m}$$

a d

$$F_2(x)=<S\ (\omega),\overline{\omega}f(\omega)_{S_m},$$

a d $S\ (\omega)$ e S e e e e ba N e a f e ba $S\ (\omega)=C\ (\omega)$. We a e a F_1 a d F_2 b a e (f -) e $B(1)$. If f a e- e ab , e F_1, F_2 be e Ha d ace $H^2(B(1))$

$$\lim_{r\to 1-}F_1(r\xi)=\mathbf{P}f(\xi)$$

a d

$$\ _{r\to 1-} F_2(r\xi) = \mathbf{P}(\overline{\varpi} f)(\xi),$$

e e **P** e a ec e a f $L^2(S_m)$ $H^2(S_m)$.

N e a e dec (2.7) a b a ed b e f dec f $P(x,\omega)$:

$$P(x,\omega) = \overline{S\ }(\omega) + x\overline{S\ }(\omega)\overline{\omega}, \quad x \in B(1), \omega \in S_m.$$

S ce f e ba $S\ (\omega) = C\ (\omega)$, a a a e c ca D b a be ed b e e e Ca c a f a .

( ) N e a (2.7) e e a e $\mathbf{R}^{m+1}$ e f e c ce e D b f e d c e c e **C** (ee [2]).

G e $f \in L^2(S_1)$, e u

$$\begin{cases} \Delta u(x) = 0 & x \in B(1) \\ u|_{S_1}(x) = f(x) & x \in S_1, \end{cases} \tag{2.8}$$

e b

$$u(z) = h(z) + \overline{H(z)},$$

e e

$$h(z) = (Sf)(z)$$

a d

$$H(z) = zS($$

e e $\mathcal{H}$ e H be a f a $\mathbf{R}^m$. N e a e e e ca e $\mathcal{H}$ a e ea a e a a a f e b da a e f a (f -) e f c e e - a ace $\mathbf{R}^{m+1}_+$. T e b ded e f e H be a f a efe ed a f e R a f a ([14]).

We e e c e d a D b b e e - a ace e de e f c

$$\sigma^{\pm} = \frac{1}{2}(1 \pm i\overline{\mathbf{e}}_0).$$

N e a e a e e f c $\alpha(x)$ a d $\beta(x)$ defi ed §1 a d e a e c a f c . T e a D b a e ed a f .

G e $u \in L^p(\mathbf{R}^m), 1 < p < \infty$, fi d W $\mathbf{R}^{m+1}_+$ c a (1.1) a d (1.2) e ec e f $\alpha(x) = \sigma^+$ a d $\beta(x) = \sigma^-$, e e $f = u, \Omega = \mathbf{R}^{m+1}_+, \Sigma = \mathbf{R}^m$.

We c a e a D b a e ed b

$$W^{\pm}(x) = C(2\sigma^{\pm}u)(x).$$

I deed, e a e $\partial W^{\pm}(x) = 0$, a d, a a a e f fac , $W^{\pm} \in H^2(\mathbf{R}^{m+1}_+)$. A f e b da c d , a

$$\lim_{x_0 \to 0+} \sigma^+ W^+(x_0, \underline{x}) = \sigma^+ u(\underline{x}),$$

e ca e f W^- be .

T e b ded e f e R a f e P e S f - . We e ef e a e

$$\lim_{x_0 \to 0+} W^+(x_0, \underline{x}) = \frac{1}{2}[2\sigma^+ u(\underline{x}) + \mathcal{H}(2\sigma^+ u)(x)].$$

N e a f $\underline{x}, \underline{y} \in \mathbf{R}^m$,

$$(1 + i\overline{\mathbf{e}}_0)(\underline{x} - \underline{y})\overline{\mathbf{e}}_0(1 + i\overline{\mathbf{e}}_0) = 0, \tag{3.1}$$

e ce

$$\sigma^+ \mathcal{H}(\sigma^+ u) = 0.$$

T e e e fac a $\sigma^+2 = \sigma^+$, e e

$$\lim_{x_0 \to 0+} \sigma^+ W^+(x_0, \underline{x}) = \sigma^+ u(\underline{x}).$$

A a ,

$$\lim_{x_0 \to 0+} \sigma^- W^-(x_0, \underline{x}) = \sigma^- u(\underline{x}).$$

N e c de a dec f e P e e e f e Ca c e e a d e c e d f e c a D b .

Defi e, f $(x, \underline{y}) \in \mathbf{R}^{m+1}_+ \times \mathbf{R}^m$, e e e f c

$$C^{\pm}(\underline{y}) = \frac{2}{A_{m+1}} \sigma^{\pm} \frac{x - \underline{y}}{|x - \underline{y}|^{m+1}} \overline{\mathbf{e}}_0 \sigma^{\pm}.$$

A a f a d c a a

$$4x_0 = (1 + i\overline{\mathbf{e}}_0)(x - \underline{y})\overline{\mathbf{e}}_0(1 + i\overline{\mathbf{e}}_0) + (1 - i\overline{\mathbf{e}}_0)(x - \underline{y})\overline{\mathbf{e}}_0(1 - i\overline{\mathbf{e}}_0),$$

e ad e f dec f e P e e $P(x,\underline{y})$ f $\mathbf{R}^{m+1}_{+}$:

$$P(x,\underline{y}) = \frac{2}{A_{m+1}} \frac{x_0}{|x-\underline{y}|^{m+1}} = C^{+}(\underline{y}) + C^{-}(\underline{y}), x \in \mathbf{R}^{m+1}_{+}; y \in \mathbf{R}^m$$

C e e , f $u \in L^p(\mathbf{R}^m)$ e ,

$$\begin{aligned} \int_{\mathbf{R}^m} P(x,\underline{y})u(\underline{y})d\underline{y} &= \int_{\mathbf{R}^m} C^{+}(\underline{y})u(\underline{y})d\underline{y} + \int_{\mathbf{R}^m} C^{-}(\underline{y})u(\underline{y})d\underline{y} \\ &= \sigma^{+}W^{+}(x) + \sigma^{-}W^{-}(x). \end{aligned}$$

A $\sigma^{+}W^{+}$ a d $\sigma^{-}W^{-}$ b a e a $\mathbf{R}^{m+1}_{+}$, a d

$$\lim_{x_0 \to 0+} \left(\sigma^{+}W^{+}(x_0,\underline{x}) + \sigma^{-}W^{-}(x_0,\underline{x})\right) = u(\underline{x}),$$

f a e (e) e c a D b

$$\begin{cases} \Delta U(x) = 0 & x \in \mathbf{R}^{m+1}_{+} \\ U|_{\mathbf{R}^m}(x) = u(x) & x \in \mathbf{R}^m, \end{cases} \tag{3.2}$$

e b

$$U(x) = \sigma^{+}W^{+}(x) + \sigma^{-}W^{-}(x).$$

T a e D b a be ed b e Ca c a f a .

A e ba ca e e c e e dec f e P e e a d e c e d dec f e f e D b (3.2) e e S e e e e e a ace.

F e a ace ca e e S e e e , a e ba ca e, e a e a e Ca c e e (ee [3]):

$$S(\underline{y}) = \frac{1}{A_{m+1}} \overline{\mathbf{e}}_0 \frac{x-\underline{y}}{|x-\underline{y}|^{m+1}} = C(\underline{y}).$$

D e e e e a e

$$(x-\underline{y})\overline{\mathbf{e}}_0 + \overline{\mathbf{e}}_0\left((x-\underline{y})\overline{\mathbf{e}}_0\right)\mathbf{e}_0 = 2x_0,$$

e b a

$$P(x,\underline{y}) = \overline{S(\underline{y})} + \overline{\mathbf{e}}_0 \overline{S(\underline{y})}\mathbf{e}_0. \tag{3.3}$$

C e d , e a e e dec f e f e D b- :

$$U(x) = F_1 + \overline{\mathbf{e}}_0 F_2, \quad x \in \mathbf{R}^{m+1}_{+} \tag{3.4}$$

e e

$$F_1(x) = \langle S, u, \quad F_2(x) = \langle S, \mathbf{e}_0 u.$$

N e a f $u \in L^2(\mathbf{R}^m)$, F_1 a d F_2 b be e Ha d ace $H^2(\mathbf{R}^{m+1}_{+})$. T e dec (3.4) a a ead b a ed [6].

4. Half Dirichlet Problems for General Domains

Le e b da face Σ f a e e a d a Ω be e b $\Phi(x) = 0$. We eca a e e de e a e defi ed b

$$\alpha(x) = \frac{1}{2}\left(1 + i\frac{\partial\Phi(x)}{|\partial\Phi(x)|}\right), \quad \beta(x) = \frac{1}{2}\left(1 - i\frac{\partial\Phi(x)}{|\partial\Phi(x)|}\right),$$

e e e ec $\partial\Phi(x)/|\partial\Phi(x)|$ a a ec e face Σ a e $x \in \Sigma$, de ed b

$$n = \frac{\partial\Phi(x)}{|\partial\Phi(x)|}.$$

F e ba ca e $\Phi(x) = \sum_{k=0}^{m} x_k 2 - 1$ a d f e e a ace ca e $\Phi(x) = x_0$. F e e ca e $n = \frac{}{||}$ a d $n = \mathbf{e}_0$, e ec e. I e f e ec e c ca e f de e e e e c e a e (2.1) a d (3.1), e ec e. If e a D b (1.1) a d (1.2) e e ab e a e a e , e e f e

$$(1 + in\ )(x - y)n\ (1 + in\ ) = 0.$$

T c a , a acc $n^2 = -1$, e e e

$$[(x - y)n\ + n\ (x - y)] + i[n\ (x - y)n\ - (x - y)] = 0,$$

$$(x - y)n\ + n\ (x - y) = 0 \quad \textit{and} \quad n\ (x - y)n\ - (x - y) = 0.$$

B n b de f e ec d e a , bec e e a e a e fi , a d e f e ed ce e e

$$< x - y, n\ + < x - y, n\ = 0, \quad \textit{and} \quad (x - y) \wedge n\ - (x - y) \wedge n\ = 0,$$

$$(x - y) \perp (n\ + n\ ) \quad \textit{and} \quad (x - y) \parallel (n\ - n\ ).$$

T e c d a e face be a e e a m-d e - a e e, a e e ac e ca e d ed e e ec . We c e ec a e a a a e e face e a e e a d a ace .

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Richard Delanghe
Department of Mathematical Analysis
Ghent University
Galglaan 2
9000 Gent
Belgium
e-mail: Richard.Delanghe@UGent.be

Tao Qian
Department of Mathematics
University of Macau
Macao SAR
P.R. China
e-mail: fsttq@umac.mo

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