Math. Meth. Appl. Sci. 29

Faculty of Science and Technology University of Macau P.O. Box 3001 Macau

ρ θ $-\rho \geqslant \theta' \geqslant$ $\rho \equiv$ $\rho \geqslant$ © &

$$\frac{}{\pi}\int_{-\infty}^{\infty} \frac{}{-}$$

Received 13 September 2005

fi

ρ θ $\rho \geqslant$

ρ θ

fi

T

$\rho \cdot \theta \cdot \rho \; \theta$

ρ θ instantaneous amplitude instantaneous phase $\theta' \geqslant \theta' \leqslant$ fi θ' fi fi instantaneous frequency U $\theta' \geqslant \theta' \leqslant$ fi

S S $\mathbb{D}$ S $\mathbb{C}$ S $\mathbb{C}$ $\mathbb{D}$ $\mathbb{C}$ $\mathbb{D}$ T fi

$$\int_{-\pi}^{\pi}$$

$$\sum_{-\infty}^{\infty} \quad \sum_{-\infty}^{\infty}$$

T T

Theorem 1.1

T ρ $\rho \geqslant$ $\rho \in$ S $\leqslant \; \leqslant \infty$ S S ρ ρ

S $\mathbb{C}$ ∞

T

T T A

$\rho \equiv$

fi

π — − T
fi T T
T

Theorem 1.2
ρ θ ρ $-\theta$

Proof
A ρ θ

$$\rho \cdot \quad \theta \cdot \quad \rho \quad \theta$$

—

$$\rho \cdot \quad \theta \cdot \quad -\rho \quad \theta$$

T

$$\rho \cdot \quad -\theta \cdot \quad -\rho \quad -\theta \qquad \rho \cdot \quad -\theta \cdot \quad \rho \quad -\theta$$

T

$$\rho \cdot \; {}^{-\theta} \cdot \quad \rho \; {}^{-\theta}$$

T ρ $-\theta$ $\rho \geqslant \; -\theta' \leqslant$
T — T
□

T
− T −
s −

T

T

U A VA U TA U T

T
$\mathbb{C}$ domain A univalent ff
ff fi

De nition 2.1

A Ω $\in \Omega$ $\in \Omega$ $\in \Omega$ A $\mathbb{D} \to \mathbb{D}$ $\mathbb{D}$

convex domains convex functions

De nition 2.2

A Ω $\in \Omega$ $-$ $\in \Omega$ $\in \Omega$ A $\mathbb{D} \to \mathbb{D}$ $\mathbb{D}$

T T

$$\cdots \qquad \cdots \quad | \, |$$

$\mathbb{D}$ T

$$\cdots \qquad \cdots \quad | \, |$$

T * * * * normalized starlike functions normalized convex functions T * A T * fi fi

De nition 2.3

ρ θ $\leqslant \; \leqslant \pi$ $\rho \geqslant$

$$\int^{\pi} \rho \quad \theta$$

$\rho \quad \theta$ $\theta \; \pi - \theta \quad \pi$ $\theta \; \pi - \theta \quad - \pi$

A T

Theorem 2.1

$\rho \quad \theta$ $\rho \quad -\theta$

T $\mathbb{D}$

T T

fi $\bar{\mathbb{D}}$ $\mathbb{D}$

′ ′ $\mathbb{D}$

ρ θ

T

T

Theorem 2.2

ρ θ $\leqslant$ $\leqslant$ π

fi

Proof

fi ρ θ T

∞ $\mathbb{D}$

θ θ θ π argument principle

T θ $\mathbb{D}$

T

ρ θ

$\mathbb{D}$ fi

∞ $\mathbb{D}$ T

ρ θ

θ

T θ θ θ π

π

T □

...

T

$$\sum^{\infty}$$

∞

...

T

∞

$\mathbb{D}$

fi

De nition 2.4

A ∞

fi

Theorem 2.3

ρ θ ⩽ ⩽ π ρ θ ⩽ ⩽ π ∞

fi

Example 2.1 The Circle Family

T A

𝔻 𝔻 ∋ π T τ

τ — ...

— | |

— τ

T 𝔻 T 𝔻 𝔻 𝔻 /

$\sqrt{\;-\;|\;|\;\;-\;\;|\;|\;}$ — — | | —

| | T | | fi T ⩽ | | 𝔻 — | | | | | |

A argument principle T T T Ω

$$\int^{\pi} \frac{}{-} \alpha$$

α

$$\int^{\pi} \alpha \quad \alpha \quad -\alpha \quad \alpha \; -$$

T α

$\mathbb{D}$

$$\left(\int^{\pi} \frac{}{- \; -} \alpha \right)$$

α T

$\mathbb{D}$ * T $\pi - \pi \approx$ radius of starlikeness $\leqslant$ T $\mathbb{D}$ $\in$ T T T $\in$ $\mathbb{D}$

V $- \sqrt{} \approx \cdots$ radius of convexity $\leqslant$ V $\mathbb{D}$ $\in$

T T

T fi T

Theorem 3.1

A $-$ ρ^{θ} $\leqslant \pi$ $\rho \in \pi$ $\leqslant \; \leqslant \infty$ T

$\leqslant \; \leqslant \infty -$ ρ^{θ} $-\infty \; \infty$ $\rho \; \theta$ $\rho \; \pi \; \rho \; \theta \; \pi \; \theta \; \pi$ $\leqslant \infty$

$$\frac{}{} \rho \quad \in \mathbb{R}$$

T T

ρ θ $-\sqrt{\ } - \pi$ $-\infty, \infty$

$\infty -$

ρ θ $-\infty, \infty$

T

Lemma 3.1

$- \in -\pi\ \pi \leqslant \leqslant \infty$ π $-$

T π $-\pi\ \pi$ $--$ fi

$$\epsilon\to \quad \to\infty\ \overline{\pi} \int_{\epsilon} |-| \quad \pi \quad \overline{-}$$

Proof

∞ ∞ ∞

$$\overline{\pi}\,\epsilon\to \quad \to\infty \int_{-\pi\,\pi\,\cap\{|x-|\ \epsilon\}} \left(\sum_{-} \frac{}{-x - \pi} \right) x \quad x$$

$$\overline{\pi}\,\epsilon\to \int_{-\pi\,\pi\,\cap\{|x-|\ \epsilon\}}$$ 43184Tm 4f β $\beta\ \beta\ \beta\ \beta$

T A

$\kappa \in \mathbb{C}$ T T

$- \sqrt{} - \pi$

$$-\left(- \left(\frac{-}{\sqrt{}} \right) - \frac{\pi}{} \right) - \frac{}{} \geqslant$$

T κ

$$-\theta \qquad \theta' \qquad \frac{}{} \geqslant$$

∞ T □

T Journal of Integral Equations and Applications 17

IEEE Transactions on Signal Processing 45

T A Physica D 203

T A Journal of Mathematical Analysis and Applications 314

et al T Proceedings of the Royal Society London Series A 454

T T

T International Journal of Wavelets Multiresolution and Information Processing 3

A Proceedings of the IEEE 51

The Bieberbach Conjecture A A

Geometric Theory of Functions of a Complex Variable

T A

Univalent Functions V

Univalent Functions

A Univalent Functions T

Bounded Analytic Functions A