澳門四高校聯合入學考試（語言科及數學科）

Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

2023 年試題及參考答案
2023 Examination Paper and Suggested Answer

數學附加卷　Mathematics Supplementary Paper

1.

1.1 22

1.2

2.

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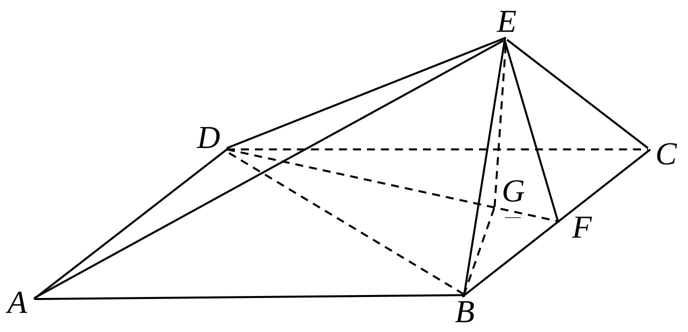
Instructions:

1. Each candidate is provided with the following documents:
    1.1 Question paper including cover page 22 pages
    1.2 One sheet of draft paper
2. Fill in your JAE No., campus, building, room and seat no. on the front page of the examination paper.
3. There are 5 questions in this paper, each carries 20 marks. Answer any 3 questions. Full mark of this paper is 60.
4. Put your answers in the lined pages provided. Answers put elsewhere will not be marked.
5. Show all your steps in getting to the answer. Full credits will be given only if the answer and all the steps are correct and clearly shown.
6. The diagrams in this examination paper are not drawn to scale.
7. Calculators of any kind are not allowed in the examination.
8. Answer the questions with a blue or black ball pen.
9. Candidates must return the question paper and draft paper at the end of the examination.

3

Answer any 3 questions, each carries 20 marks. Write down the answers on the 3 lined pages following each question.

1.



E-ABCD *ABCD*

F *BC* *G* *E* *DF*

(a) [:] (8)

(b) *EG* *ABCD* [:] (7)

(c) (5)

In the above figure, *E-ABCD* is a pyramid, its base *ABCD* is a rhombus,

and . , , *F* is the midpoint of *BC*, *G* is the foot of

perpendicular from *E* to *DF*. .

(a) Show that [Hint. Find and .] (8 marks)

(b) Show that *EG* is perpendicular to plane *ABCD*.

[Hint. Show that is a right-angled triangle.] (7 marks)

(c) Find . (5 marks)

2. (a) ◦

(i) ◦ (2)

(ii) $f(x)$ ◦ (3)

(iii) ◦ (2)

(iv) (i) (iii) (3)

(b) 3 *A*

(i) *A* ◦ (4)

(ii) (6)

(a) Given function .

(i) Find and . (2 marks)

(ii) Find the local maximum and local minimum values of (3 marks)

(iii) Find the inflection point(s) of the curve . (2 marks)

(iv) Using the results in (i) (iii), sketch the curve (3 marks)

(b) Given that the line is a tangent line of the curve 3 at point *A*.

(i) Find the point *A*. (4 marks)

(ii) Find the area of the region bounded by the line and the curve . (6 marks)

3. E: — — E L_1 L_2 A

4.

(a)

(8)

(b) (i)

and

(5)

(ii) (i) tan (7)

Let .

(a) Let , where *x* and *y* are real numbers. If satisfies the equation , find the value of . (8 marks)

(b) (i) Using G theorem, show that

and .

Deduce that . (5 marks)

(ii) Using the result in (i), solve the equation .

Express your answer in terms of tan. (7 marks)

5. (a) (i)

cos —— sin —— sin —— sin —— (4)

(ii) (6)

(b) – x y z

–

(10)

(a) (i) Given the formulas

and

…꞉ ‘3 “ Q E % $íaÜN A ™Ḿ… .

Deduce that

cos —— sin —— and sin —— sin ——. (4 marks)

(ii) Prove the identity

) … ‘ – ‘ ™ ž Å › $ > v

1. (a) F BC

F BC

(b)

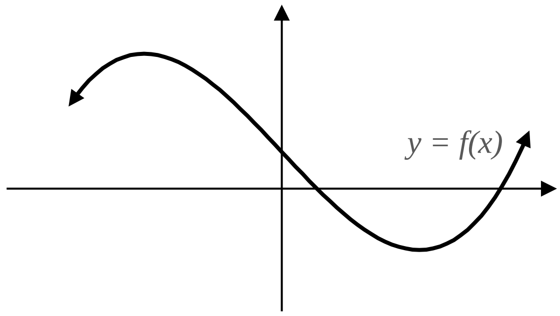
(c) F BC

AG

2. (a)(i)

(ii)

(iv)



(b)(i) L 1 A —

L C

, L C A

(ii) ((i))

,

–

—

3. (a)(i) — —

(1)

E (1) 0

[

(2)

(ii) (2)

(3)

(3) —— ——

(b)

——

A

(c)

4. (a)

(b) (i)

(ii)

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5. (a)(i)

(a)(ii)

(b) (a)(ii)

Suggested Answers:

1. (a) From *F* is the midpoint of *BC* and , we get and – –.

Then, – – .

From – and , we get is equilateral. So, .

From *F* is the midpoint of *BC* and , we get .

Then, .

Hence, .

(b) In ,

In ,

In , .

Since , we have –.

From and , we get .

(c) From *F* is the midpoint of *BC* and , we get – –.

Join *A* and *G*. Since – – –, we have .

Since – – –,

we have .

Hence,

(iv)

(b)(i) The slope of line L is 1. At A,

(c) For let be the slope of , where – –.

As , we have – and –.

Suppose the angle between and is , where –. Then,

Hence, the angle between and is .

4. (a)

From the second equation, . Solving the first equation with , we have

Hence, .

(b) (i)

Comparing the real and imaginary parts, we get

and .

Hence,

(ii) Let , – –. Then,

– is an integer

– is an integer.

Since – –, the roots of the equation are – and .

[Remark: If we let – or – , then the values of are –, and .

The answers are the same because .]

5. (a)(i)

(a)(ii)

(b) Since the system of equations has more than one solution, using the result in (a)(ii), we have

.

As –, we have or – or –.

If or –, the second equation becomes –, from which we know that the system of equations has no solution. Hence, and –.

When – the system of equations becomes .

Solving –, we get , .