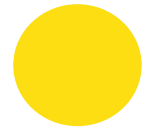


門大



( )

**Joint Admission Examination for Macao Four Higher Education Institutions  
(Languages and Mathematics)**

**2023  
2023 Examination Paper and Suggested Answer**

**Mathematics Standard Paper**

第一部份 選擇題。請選出每題之

1.  $M = \{x \mid x^2 - 2x - 8 \geq 0\}$   $N = \{x \mid 0 < x < 6\}$   $M \cap N = ( \quad )$   
 A.  $[-2, 4]$                       B.  $[-2, 0)$                       C.  $(0, 4]$                       D.  $(0, 6)$                       E.  $[4, 6)$
2.  $f(x) = x^2 - x - 6$   $3x - 2$   $f(3) = ( \quad )$   
 A.  $-2$                       B.  $0$                       C.  $3$                       D.  $7$                       E.  $9$
3.  $\log_9 125 \times \log_{12} 17 \times \log_{25} 3 \times \log_{17} 12 = ( \quad )$   
 A.  $\log_{17} 3$                       B.  $\frac{1}{2}$                       C.  $\frac{3}{4}$                       D.  $\log_3 35$                       E.  $\log_{17} 12$
4.  $x^2 - 3x + 4\sqrt{x^2 - 3x} = 12$   $( \quad )$   
 A.  $\{-1\}$                       B.  $\{2, -6\}$                       C.  $\{-1, 4\}$                       D.  $\{4\}$                       E.  $\{3\}$
5.  $a$   $4a^2x^2 + 2(a+3)x + 9 = 0$   $a = ( \quad )$   
 A.  $\frac{3}{5}$                       B.  $-1$                       C.  $\frac{3}{2}$                       D.  $-\frac{3}{7}$                       E.  $\frac{3}{5}$
6.  $\left(2\sqrt{x} - \frac{1}{\sqrt{x}}\right)^6$   $( \quad )$   
 A.  $-8$                       B.  $8$                       C.  $-160$                       D.  $160$                       E.  $1$
7.  $f(x) = ax^2 + 4x + 1$   $a \in \mathbb{R}$   $(2, 4)$   $a$   $( \quad )$   
 A.  $\left[-\frac{1}{2}, 0\right)$                       B.  $\left(0, \frac{1}{2}\right]$                       C.  $\left[-\frac{1}{2}, \frac{1}{2}\right]$                       D.  $\left[-\frac{1}{2}, \infty\right)$                       E.  $\left[\frac{1}{2}, \infty\right)$
8.  $f(x) = \begin{cases} \log_2 x, & 0 < x \leq 4 \\ x^2 - 8x + 17, & x > 4 \end{cases}$   $f\left(\frac{1}{2} - 3|x|\right) + f(5) > 0$   $( \quad )$   
 A.  $-\frac{1}{12} < x < \frac{1}{12}$                       B.  $-\frac{1}{6} < x < \frac{1}{6}$                       C.  $-\frac{1}{4} < x < \frac{1}{4}$   
 D.  $-\frac{1}{3} < x < \frac{1}{3}$                       E.  $-\frac{1}{2} < x < \frac{1}{2}$

9.  $3^3 \cdot 8^5 \cdot 2^2$  ( )

- A.  $\frac{2}{3}$       B.  $\frac{3}{2}$       C. 1      D.  $\frac{16}{27}$       E.  $\frac{32}{27}$

10.  $7^7 \cdot 80^{16} \cdot 26^{26} \cdot 34^{34}$  ( )

A. -6      B. -82      C. -88      D. -198      E. -204

11.  $A(3, -8)$   $B(-7, 4)$   $AB$   $3x - 4y + 14 = 0$  ( )

A.  $4x + 3y + 14 = 0$       B.  $3x + 4y + 14 = 0$       C.  $3x - 4y - 14 = 0$

D.  $4x - 3y + 14 = 0$       E.  $4x + 3y - 14 = 0$

12.  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  ( $a, b > 0$ )  $3 \frac{b^2 + 2}{a}$  ( )

A. 2      B.  $2\sqrt{2}$       C.  $2\sqrt{3}$       D. 4      E. 8

13.  $A$   $B$   $\sin A = \frac{2}{5}$   $\sin B = \frac{4}{5}$   $\sin(A + B) =$  ( )

A.  $\frac{-6 - 4\sqrt{21}}{25}$       B.  $\frac{13}{25}$       C.  $\frac{18}{25}$

D.  $\frac{-12 - 2\sqrt{21}}{25}$       E.  $\frac{12 + 2\sqrt{21}}{25}$

14.  $y = a \sin(x - \frac{\pi}{6}) + b$   $a$   $b$  ( )

- A.  $a = -4$   $b = 4$       B.  $a = -2$   $b = 2$
- C.  $a = 2$   $b = -2$       D.  $a = 4$   $b = -4$
- E.

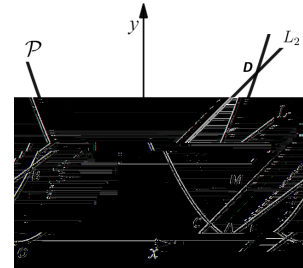


15.  $A(-2, 3)$   $O$   $90^\circ$   $B$

第二部份 解答題。

1.  $\frac{1}{4}$
- (a) (3 )
- (b) (2 )
- (c) (3 )

2.  $\mathcal{P}: x^2 = 4y$   $F$   $F$
- $\frac{3}{4}$   $L_1$   $\mathcal{P}$   $A$   $B$
- 1  $L_2$   $\mathcal{P}$   $C$   $D$   $y$
- $M$



- (a)  $F$  (2 )
- (b)  $AB$  (3 )
- (c)  $|DM| = 3|CM|$   $CD$  (3 )
3.  $S_n = 3^{n+1} - 2k$   $\{a_n\}_{n \geq 1}$   $n$   $k \in \mathbb{R}$

- (a)  $k$   $a_n$  (3 )
- (b)  $b_n = \frac{1}{a_n} + \log_2 a_n$   $b_n$   $n$   $T_n$  (3 )
- (c)  $c_n = \frac{2}{a_n}$   $f(n) = -5c_n^2 + c_n$   $n$  (2 )

4.  $f(x) = \sqrt{3} \sin(2wx) - 2\cos^2(wx)$   $3\pi$
- (a)  $f(x)$  (4 )
- (b)  $\triangle ABC$   $f(C) = 0$   $2\sin^2 B = \cos B + \cos(A - C)$   $\sin A$  (4 )

5.  $x, y$   $\begin{cases} 3x + 2y - 13 \geq 0 \\ x \leq 5 \\ 2x - 2y + 3 \geq 0 \end{cases}$
- (a) (2 )
- (b)  $z = \frac{y}{x}$   $z$  (3 )
- (c)  $t = x^2 + y^2$   $t$  (3 )

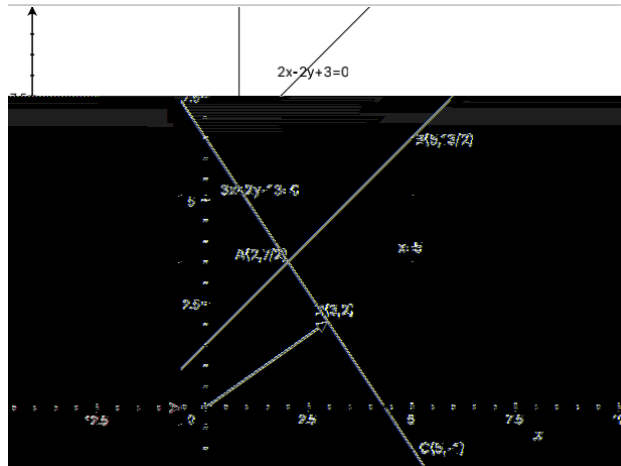
第一部份 選擇題。

1	E
2	D
3	C
4	C
5	D
6	C
7	D
8	A
9	E
10	B
11	A
12	E
13	A
14	B
15	E



$$\begin{aligned}
 \text{(b)} \quad f(C) &= 2 \sin\left(\frac{2}{3}C - \frac{\pi}{6}\right) - 1 = 0 & \sin\left(\frac{2}{3}C - \frac{\pi}{6}\right) &= \frac{1}{2} & -\frac{\pi}{6} < \frac{2}{3}C - \frac{\pi}{6} < \frac{\pi}{2} \\
 \frac{2}{3}C - \frac{\pi}{6} &= \frac{\pi}{6} & C &= \frac{\pi}{2} & A + B &= \frac{\pi}{2} & 2\sin^2 B &= \cos B + \cos\left(A - \frac{\pi}{2}\right) \\
 2\sin^2 B &= \sin A + \sin A & \sin^2 B &= \sin A & 1 - \sin^2 A &= \sin A & \sin A &= \frac{\sqrt{5}-1}{2}
 \end{aligned}$$

5. (a)



$$\begin{aligned}
 \text{(b)} \quad y/x & \quad P(x, y) & A(2, 7/2), B(5, 13/2) \\
 C(5, -1) & \quad P & z & \quad C & \quad A \\
 -1/5 \leq z & \leq 7/4
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad t & \quad P(x, y) & D & \quad 3x + 2y - 13 = 0 \\
 OD & & OD & \quad 2/3 & \quad y = 2x/3 & \quad OD \\
 3x + 2y - 13 = 0 & \quad (x, y) = (3, 2) & t & \quad (x, y) = (3, 2) & \quad 13
 \end{aligned}$$

1. Let  $M = \{x \mid x^2 - 2x - 8 \geq 0\}$  and  $N = \{x \mid 0 < x < 6\}$ , then  $M \cap N = ( \quad )$ .
- A.  $[-2, 4]$                       B.  $[-2, 0)$                       C.  $(0, 4]$                       D.  $(0, 6)$                       E.  $[4, 6)$
2. If we divide the polynomial  $f(x)$  by  $x^2 - x - 6$  and the remainder is  $3x - 2$ , then  $f(3) = ( \quad )$ .
- A.  $-2$                       B.  $0$                       C.  $3$                       D.  $7$                       E.  $9$
3.  $\log_9 125 \times \log_{12} 17 \times \log_{25} 3 \times \log_{17} 12 = ( \quad )$ .
- A.  $\log_{17} 3$                       B.  $\frac{1}{2}$                       C.  $\frac{3}{4}$                       D.  $\log_3 35$                       E.  $\log_{17} 12$
4. The set of solutions for the equation  $x^2 - 3x + 4\sqrt{x^2 - 3x} = 12$  is  $( \quad )$ .
- A.  $\{-1\}$                       B.  $\{2, -6\}$                       C.  $\{-1, 4\}$                       D.  $\{4\}$                       E.  $\{3\}$
5. Let  $a$  be a constant and suppose the quadratic equation  $4a^2x^2 + 2(a + 3)x + 9 = 0$  has exactly one real solution. Then  $a = ( \quad )$ .
- A.  $\frac{3}{5}$                       B.  $-1$  or  $\frac{3}{2}$                       C.  $\frac{3}{2}$   
D.  $-\frac{3}{7}$  or  $\frac{3}{5}$                       E. any real number
6. The constant term in the expansion of  $\left(2\sqrt{x} - \frac{1}{\sqrt{x}}\right)^6$  is  $( \quad )$ .
- A.  $-8$                       B.  $8$                       C.  $-160$                       D.  $160$                       E.  $1$
7. The function  $f(x) = ax^2 + 4x + 1$  ( $a \in \mathbb{R}$  is a constant) is increasing on the open interval  $(2, 4)$ . Then the range of  $a$  is  $( \quad )$ .
- A.  $\left[-\frac{1}{2}, 0\right)$                       B.  $\left(0, \frac{1}{2}\right]$                       C.  $\left[-\frac{1}{2}, \frac{1}{2}\right]$                       D.  $\left[-\frac{1}{2}, \infty\right)$                       E.  $\left[\frac{1}{2}, \infty\right)$
8. Let  $f(x) = \begin{cases} \log_2 x, & 0 < x \leq 4 \\ x^2 - 8x + 17, & x > 4 \end{cases}$ . The solution of the inequality  $f\left(\frac{1}{2} - 3|x|\right) + f(5) > 0$  is  $( \quad )$ .
- A.  $-\frac{1}{12} < x < \frac{1}{12}$                       B.  $-\frac{1}{6} < x < \frac{1}{6}$                       C.  $-\frac{1}{4} < x < \frac{1}{4}$   
D.  $-\frac{1}{3} < x < \frac{1}{3}$                       E.  $-\frac{1}{2} < x < \frac{1}{2}$

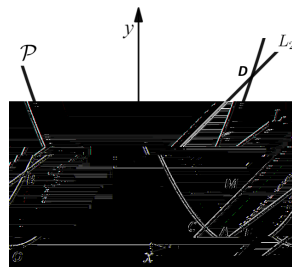


9. An upright cylindrical water tank has an inner radius of 3 meters and a height of 8 meters, and the current water depth is 5 meters. If a sphere with a radius of 2 meters is placed into the water tank and the sphere is completely immersed in the water, the water level rises by ( ) meters.
- A.  $\frac{2}{3}$                       B.  $\frac{3}{2}$                       C. 1                      D.  $\frac{16}{27}$                       E.  $\frac{32}{27}$
10. In an arithmetic sequence, the 7th term is 80 and the 16th term is 26. Then the 34th term is ( ).
- A. -6                      B. -82                      C. -88                      D. -198                      E. -204
11. Let  $A$  and  $B$  be the points  $(3, -8)$  and  $(-7, 4)$  respectively. An equation of the line passing through the midpoint of  $AB$  and perpendicular to  $3x - 4y + 14 = 0$  is ( ).
- A.  $4x + 3y + 14 = 0$                       B.  $3x + 4y + 14 = 0$                       C.  $3x - 4y - 14 = 0$
- D.  $4x - 3y + 14 = 0$                       E.  $4x + 3y - 14 = 0$
12. If the eccentricity of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  ( $a, b > 0$ ) is 3, then the minimum value of  $\frac{b^2 + 2}{a}$  is ( ).
- A. 2                      B.  $2\sqrt{2}$                       C.  $2\sqrt{3}$                       D. 4                      E. 8
13. Let  $A$  and  $B$  be angles in the second quadrant such that  $\sin A = \frac{2}{5}$  and  $\sin B = \frac{4}{5}$ . Then  $\sin(A + B) =$  ( ).
- A.  $\frac{-6 - 4\sqrt{21}}{25}$                       B.  $\frac{13}{25}$                       C.  $\frac{18}{25}$
- D.  $\frac{-12 - 2\sqrt{21}}{25}$                       E.  $\frac{12 + 2\sqrt{21}}{25}$
14. The right figure shows the graph of  $y = a \sin(x - \frac{\pi}{6}) + b$ , where  $a$  and  $b$  are constants. Then ( ).
- A.  $a = -4$  and  $b = 4$                       B.  $a = -2$  and  $b = 2$
- C.  $a = 2$  and  $b = -2$                       D.  $a = 4$  and  $b = -4$
- E. none of the above
- 
15. Point  $A(-2, 3)$  is rotated  $90^\circ$  clockwise about the origin  $O$  to get point  $B$ . Points  $C$  and  $B$  are symmetrical about the  $x$ -axis. Point  $C$  is translated downward three units to get point  $D$ . Then the coordinates of point  $D$  are ( ).
- A.  $(-3, -1)$                       B.  $(-3, 0)$                       C.  $(-4, 0)$                       D.  $(2, 0)$                       E.  $(3, -5)$

1. A coin is unfair that the probability of a head facing up is  $\frac{1}{4}$ .

- (a) Find the probability of obtaining at most one head facing up in 10 successive tosses. (3 marks)
- (b) Find the probability that the 10th toss will be the first of obtaining head facing up. (2 marks)
- (c) Find the probability that the 10th toss will be the third of obtaining head facing up. (3 marks)

2. In the right figure, the parabola  $\mathcal{P} : x^2 = 4y$  has its focus  $F$ . The straight line  $L_1$  of slope  $\frac{3}{4}$  passing through the focus  $F$  intersects the parabola  $\mathcal{P}$  at points  $A$  and  $B$ . Another straight line  $L_2$  of slope 1 intersects the parabola  $\mathcal{P}$  at points  $C$  and  $D$ , and intersects the  $y$ -axis at point  $M$ .



- (a) Find the coordinates of the focus  $F$ . (2 marks)
- (b) Find the length of segment  $AB$ . (3 marks)
- (c) If  $|DM| = 3|CM|$ , find the length of segment  $CD$ . (3 marks)

3. Let  $S_n = 3^{n+1} - 2k$  be the  $n$ th sum of the geometric sequence  $\{a_n\}_{n \geq 1}$ . Here  $k \in \mathbb{R}$  is a constant.

- (a) Find  $k$  and  $a_n$ . (3 marks)
- (b) Let  $b_n = \frac{1}{a_n} + \log_2 a_n$ . Find the sum  $T_n$  of the first  $n$  terms for the sequence  $b_n$ . (3 marks)
- (c) Let  $c_n = \frac{2}{a_n}$ . Find  $n$  where  $f(n) = -5c_n^2 + c_n$  obtains its maximum value. (2 marks)

4. The minimal positive period of the function  $f(x) = \sqrt{3} \sin(2wx) - 2\cos^2(wx)$  is  $3\pi$ .

- (a) Find the expression of  $f(x)$ . (4 marks)
- (b) In  $\triangle ABC$ , if  $f(C) = 0$  and  $2\sin^2 B = \cos B + \cos(A - C)$ , find the value of  $\sin A$ . (4 marks)

5. Let  $x, y$  satisfy 
$$\begin{cases} 3x + 2y - 13 \geq 0 \\ x \leq 5 \\ 2x - 2y + 3 \geq 0 \end{cases} .$$

- (a) Sketch the region satisfying the above system of inequalities. (2 marks)
- (b) Let  $z = \frac{y}{x}$ . Find the range of  $z$ . (3 marks)
- (c) Let  $t = x^2 + y^2$ . Find the minimum value of  $t$ . (3 marks)

## Suggested Answer

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Question Number	Best Answer
1	E
2	D
3	C
4	C
5	D
6	C
7	D
8	A
9	E
10	B
11	A
12	E
13	A
14	B
15	E

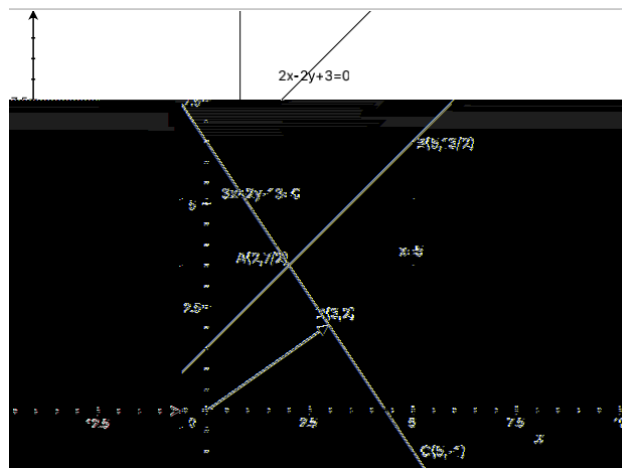
1. (a) If a coin is tossed 10 times consecutively, the probability of getting tails 10 times is  $(1 - \frac{1}{4})^{10} = (\frac{3}{4})^{10}$ .  
The probability of getting exactly one head in ten consecutive tosses of a coin is  ${}_{10}C_1 \frac{1}{4} (1 - \frac{1}{4})^{10-1} = \frac{5}{2} (\frac{3}{4})^9$ . Therefore, the probability of at most one head up is  $(\frac{3}{4})^{10} + \frac{5}{2} (\frac{3}{4})^9 = \frac{13}{4} \times (\frac{3}{4})^9$ .
- (b) The probability that the first 9 tosses are tails and the 10th toss is head is  $(1 - \frac{1}{4})^{10-1} \times \frac{1}{4} = \frac{3^9}{4^{10}}$ .
- (c) The probability of getting exactly two heads in the first 9 tosses is  ${}_{9}C_2 (\frac{1}{4})^2 (1 - \frac{1}{4})^{9-2}$ . Therefore, the probability of getting a third head on the tenth toss is  ${}_{9}C_2 (\frac{1}{4})^2 (1 - \frac{1}{4})^{9-2} \times \frac{1}{4} = (\frac{3}{4})^9$ .
2. (a) The coordinates of the focus  $F$  is  $(0, 1)$ .
- (b) The equation of the straight line  $L_1$  is  $y = \frac{3}{4}x + 1$ . Combining the equations of the straight line  $L_1$  and the parabola  $\mathcal{P}$ , we can get  $x^2 = 3x + 4$ . Solving the quadratic equation, one has  $x = -1$  or  $4$ . Then the coordinates of points  $A$  and  $B$  are  $A(-1, \frac{1}{4})$  and  $B(4, 4)$ , respectively. Then according to the definition of parabolas, we have  $|AB| = \frac{25}{4}$ .
- (c) Suppose the equation of the straight line  $L_2$  is  $y = x + t$ . Suppose the  $x$ -coordinates of points  $C$  and  $D$  are  $x_1$  and  $x_2$ , respectively. Combining the equations of the straight line  $L_2$  and the parabola  $\mathcal{P}$ , we can get  $x^2 - 4x - 4t = 0$ . Using Weda's Theorem,  $x_1 + x_2 = 4$ . Since  $|DM| = 3|CM|$ , we have  $x_2 = -3x_1$ . Furthermore, we can get  $x_1 = -2, x_2 = 6$ . Thus,  $|CD| = \sqrt{1^2 + 1}|x_2 - x_1| = 8\sqrt{2}$ .
3. (a) From the question, we get  $a_1 = S_1 = 9 - 2k, a_2 = S_2 - S_1 = 27 - 9 = 18$  and  $a_3 = S_3 - S_2 = 54$ . Since  $\{a_n\}_{n \geq 1}$  is a geometric sequence, we have  $a_1 a_3 = a_2^2$ , which implies that the first term  $a_1 = 6$ , the common ratio  $q = 3$  and  $k = 3/2$ . Then we can get  $a_n = a_1 \times q^{n-1} = 2 \times 3^n$ .
- (b) Since  $b_n = \frac{1}{2 \times 3^n} + \log_2(2 \times 3^n) = \frac{1}{2 \times 3^n} + 1 + n \log_2 3, T_n = \frac{1}{4} \left(1 - \frac{1}{3^n}\right) + n + \frac{n(n+1)}{2} \log_2 3$ .
- (c) Since  $c_n = 3^{-n}, f(n) = -5(3^{-n})^2 + 3^{-n} = -5\left(\frac{1}{3^n} - \frac{1}{10}\right)^2 + \frac{1}{20}$ . When  $n = 2$ , the function  $f(n)$  obtains its maximum value  $\frac{4}{81}$ .
4. (a) By the double-angle formula,  $f(x) = \sqrt{3} \sin 2wx - (1 + \cos 2wx)$ . So  $f(x) = 2\left(\frac{\sqrt{3}}{2} \sin 2wx -$

$\frac{1}{2} \cos 2wx) - 1 = 2 \sin(2wx - \theta) - 1$ , with  $\sin \theta = \frac{1}{2}$  and  $\cos \theta = \frac{\sqrt{3}}{2}$ . Thus  $\theta = \frac{\pi}{6} + 2k\pi$ . Since the least period of  $f(x)$  is  $\frac{2\pi}{2w} = 3\pi$ , we get  $2w = \frac{2}{3}$  and can write  $f(x) = 2 \sin(\frac{2}{3}x - \frac{\pi}{6}) - 1$ .

(b) Since  $f(C) = 2 \sin(\frac{2}{3}C - \frac{\pi}{6}) - 1 = 0$ , we have  $\sin(\frac{2}{3}C - \frac{\pi}{6}) = \frac{1}{2}$ . By observing that  $-\frac{\pi}{6} < \frac{2}{3}C - \frac{\pi}{6} < \frac{\pi}{2}$ , we can get  $\frac{2}{3}C - \frac{\pi}{6} = \frac{\pi}{6}$ . Then  $C = \frac{\pi}{2}$  and  $A + B = \frac{\pi}{2}$ . We have  $2\sin^2 B = \cos B + \cos(A - \frac{\pi}{2})$  which implies that  $2\sin^2 B = \sin A + \sin A$ . Then  $1 - \sin^2 A = \sin A$ , and  $\sin A = \frac{\sqrt{5} - 1}{2}$ .

5. Answer:

(a)



(b)  $y/x$  is the slope of straight line jointing point  $P(x, y)$  and the origin. Intersections of the given straight lines are  $A(2, 7/2)$ ,  $B(5, 13/2)$  and  $C(5, -1)$ . When point  $P$  varies inside the given region, the minimum value of  $z$  can be obtained at point  $C$  and the maximum value can be obtained at point  $A$ . Then  $-1/5 \leq z \leq 7/4$ .

(c)  $t$  is the square of the distance between point  $P(x, y)$  in the given region and the origin. The nearest point to the origin is the point  $D$  lying on the line  $3x + 2y - 13 = 0$  and  $OD$  is perpendicular to this straight line. The slope of the line  $OD$  should be  $2/3$  and the equation is  $y = 2x/3$ . Thus, the intersection of lines  $OD$  and  $3x + 2y - 13 = 0$  is  $(x, y) = (3, 2)$ . Therefore,  $t$  obtains its minimum value 13 at point  $(x, y) = (3, 2)$ .