1.

A. {0}　　B. { : sin cos 3}　　C. { : 1 0, ℝ}

D. { }　　E. {(,): 0, ℝ, ℝ}

2. > > 0 > 0

A. —　　B. —　　C. —

D. —　　E.

3. 3 2 1

A. 4　　B. 3　　C. 4　　D. 9　　E. 6

4. $\sqrt{7 \quad 4\sqrt{3}}$

A. $\sqrt{3} \quad \sqrt{2}$　　B. $2 \quad \sqrt{3}$　　C. $\sqrt{3} \quad 2$

D. $2 \quad \sqrt{6}$　　E. $2 \quad 2\sqrt{3}$

5. 2(3) 1 0

A. 0 $\frac{5}{7}$　　B. $\frac{5}{7}$　　C. $\frac{5}{7}$ 1

D. $\frac{9}{7}$　　E.

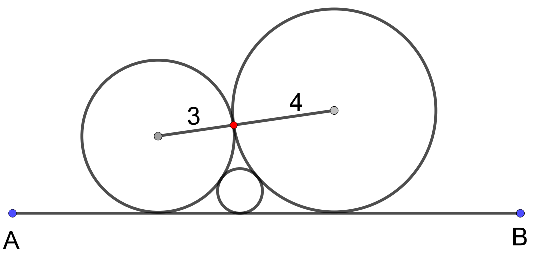
6. 2 5 $\sqrt{10}$ $\frac{1}{\quad}$ $\frac{1}{\quad}$

A. 2　　B. 1　　C. $\sqrt{2}$　　D. $\frac{\sqrt{2}}{2}$　　E. $\frac{1}{2}$

7. AB 3 4

A. $84 \quad 48\sqrt{3}$　　B. $\sqrt{2} \quad 1$　　C. $2\sqrt{2} \quad 2$

D. $6 \quad 4\sqrt{2}$　　E. $42 \quad 24\sqrt{3}$



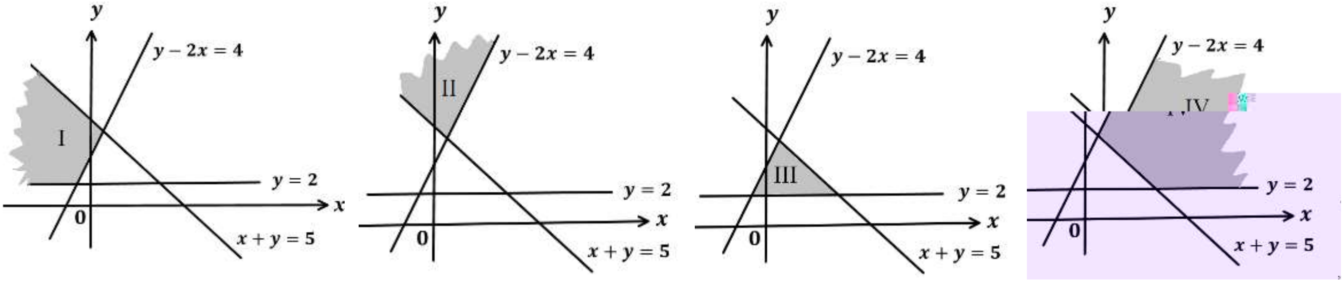
8. $\frac{1}{4}$

A. $\frac{1}{64}$　　B. $\frac{27}{64}$　　C. $\frac{35}{64}$　　D. $\frac{37}{64}$　　E. $\frac{43}{64}$

9. 2 4

5

2 0



A. B. C. III D. IV E.

10. $(1, 2)$ 2 3 4

A. $0, \frac{7}{2}$ B. $0, \frac{5}{2}$ C. $(0, 3)$ D. $0, \frac{1}{2}$ E. $0, \frac{3}{2}$

11. 6400 32

4

A. 5000 B. 5200 C. 5400 D. 5600 E. 5800

12. 3 8 0 $\frac{1}{\ }$ $\frac{1}{\ }$ 2

A. 2 B. 1 C. 4 D. 1 E. 2

13. 1, 2, 3, 4, 5 40

A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{3}{5}$ D. $\frac{4}{5}$ E. 1

14. 6 2 2 6 2

A. 1 B. $\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1

15. 150 3

$\sqrt{3}$

A. $3\sqrt{3}$ B. $\frac{\sqrt{3}}{3}$ C. $\frac{1}{3}$ D. 3 E. $2\sqrt{3}$ $\sqrt{3}$

1. ( ) (5, 0) 2 ( ) 9

(a) (3)

(b) 3 3 (2)

(c) ( ) (3sin) ( )

1	B
2	C
3	E
4	B
5	D
6	A
7	A
8	D
9	D
10	A
11	B
12	E
13	C
14	C
15	E

( )

1. (a)

25 5 0

4

4 2 9

1, 4, 5

( ) 4 5

(b) (1) 6 2 5

(c) ( ) (3sin) 4(3sin) 5 (3sin 2) 9

3 3sin 3 2 [3,3] ( ) 3sin 3

Max ( ) (3) 16

( ) 3sin 2 ,

Min ( ) (2) 9

2. (a) ,4 ,16 16 8

16 8 1 $\frac{1}{4}$

$\frac{1}{4}$ $\frac{}{4}$

(b)

$\frac{4}{3}$ 1 $\frac{1}{4}$

$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\cdots$ $\frac{1}{4}$ $\frac{}{4}$ (1)

4 1 $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\cdots$ $\frac{}{4}$ (2)

(2) (1)

3 1 $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\cdots$ $\frac{1}{4}$ $\frac{}{4}$ $\frac{4}{3}$ 1 $\frac{1}{4}$ $\frac{}{4}$

$\frac{4}{9}$ 1 $\frac{1}{4}$ $\frac{}{3\ 4}$

3. (a)

$$\sqrt{\qquad} \quad \frac{\sqrt{2}}{2}$$

$$\frac{8}{} \quad \frac{16}{} \quad 1$$

32 16

$$\frac{}{16} \quad \frac{}{32} \quad 1$$

(b) $\frac{}{16} \quad \frac{}{32} \quad 1$

$$\frac{}{16} \quad \frac{(\quad)}{32} \quad 1$$

(2) 2 32 0

(,)

$$\frac{}{2} \qquad \frac{2}{2}$$

$$\frac{2}{}$$

2

4. (a) $\sqrt{3}\cos \quad \sin \quad 2(\frac{\sqrt{3}}{2}\cos \quad \frac{1}{2}\sin \quad) \quad 2(\cos\frac{}{6}\cos \quad \sin\frac{}{6}\sin \quad) \quad 2\cos(\quad \frac{}{6})$

2 $\sqrt{3}\cos$ sin 2

(b) (a) $\sqrt{3}\cos \quad \sin \quad 2\cos(\quad \frac{}{6})$

$\sqrt{3}\cos \quad \sin \quad 1 \quad \cos(\quad \frac{}{6}) \quad \frac{1}{2}$

0 2 $\frac{}{6} \quad \frac{2}{3} \quad \frac{4}{3} \qquad \frac{}{2} \quad \frac{7}{6}$

(c) $\sqrt{3}\cos \quad \sin \quad \frac{1}{2} \quad 2\cos(\quad \frac{}{6}) \quad \frac{1}{2} \quad \cos(\quad \frac{}{6}) \quad \frac{1}{4}$

$1 \quad 2\sin(\frac{}{2} \quad \frac{}{12}) \quad \frac{1}{4} \qquad \sin(\frac{}{2} \quad \frac{}{12}) \quad \frac{5}{8} \qquad 0$

$0 \quad \frac{}{2} \quad \frac{}{12} \qquad \sin(\frac{}{2} \quad \frac{}{12}) \quad \sqrt{\frac{5}{8}} \quad \frac{\sqrt{10}}{4}$

5. ( ) 3 5 14

(1) 1 3 5 3 5 729 125 854 [illegible] 14 16 (1)

(2) (ℤ) ( ) 3 5 1[illegible]

1(ℤ)

3 5

3 5

3 3 5 5

81 3 25 5

56 3 25 3 25 5

=56 3 25 3 5

1. Which of the following is an empty set?

A. {0}　　B. { :sin　cos　3}　　C. { :　1　0,　ℝ}

D. {　}　　E. {(,): 　0,　ℝ,　ℝ}

2. If　>　> 0 and　> 0, which of the following inequalities is true?

A. —　——　　B. —　——　　C. —　——

D. —　——　　E. none of the above

3. If the polynomial　3　　is divided by　2 and　1 respectively, the remainders are equal. Find the value of .

A. 4　　B. 3　　C. 4　　D. 9　　E. 6

4. $\sqrt{7\ \ 4\sqrt{3}}$

A. $\sqrt{3}\ \ \sqrt{2}$　　B. $2\ \ \sqrt{3}$　　C. $\sqrt{3}\ \ 2$

D. $2\ \ \sqrt{6}$　　E. $2\ \ 2\sqrt{3}$

5. If the equation　2(　3)　1　0 has real roots, find the range of .

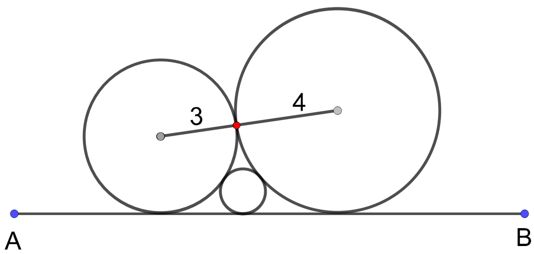
A. $0\ \ \frac{5}{7}$　　B. $\frac{5}{7}$　　C. $\frac{5}{7}\ \ 1$

D. $\frac{9}{7}$　　E. none of the above

6. Given 2　5　$\sqrt{10}$, what is the value of $\frac{1}{\ }\ \frac{1}{\ }$?

A. 2　　B. 1　　C. $\sqrt{2}$　　D. $\frac{\sqrt{2}}{2}$　　E. $\frac{1}{2}$

7. In the right figure, the three circles and the line segment AB touch each other. The two larger circles have radii 3 units and 4 units, respectively. Find the radius of the smallest circle.



A. $84\ \ 48\sqrt{3}$　　B. $\sqrt{2}\ \ 1$　　C. $2\sqrt{2}\ \ 2$

D. $6\ \ 4\sqrt{2}$　　E. $42\ \ 24\sqrt{3}$

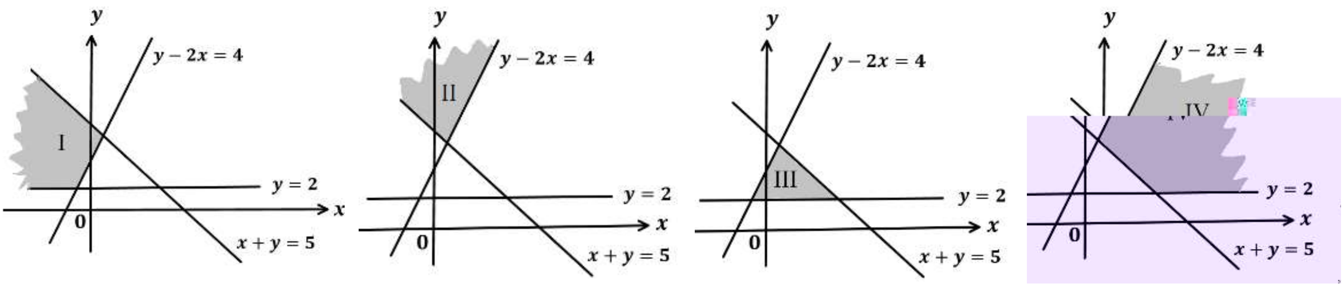
8. John has probability $\frac{1}{4}$ of winning a game. What is the probability that he wins at least one game in three consecutive games?

A. $\frac{1}{64}$　　B. $\frac{27}{64}$　　C. $\frac{35}{64}$　　D. $\frac{37}{64}$　　E. $\frac{43}{64}$

2 4

9. In the below figures, which of the regions is the solution set to the system of inequalities 5 ?

2 0



A. B. C. III D. IV E. none of the above

10. Suppose that the line passes through the point (1, 2), and is perpendicular to the line 2 3 4. and the -axis will intersect at

A. $0, \frac{7}{2}$ B. $0, \frac{5}{2}$ C. (0, 3) D. $0, \frac{1}{2}$ E. $0, \frac{3}{2}$

11. There are 6400 chairs in an indoor stadium. Each row of the venue has 32 chairs. Current social distancing measure requires that no more than 4 consecutive chairs could be occupied in the same row. To comply with this requirement, the maximum number of chairs that could be occupied in each performance is

A. 5000 B. 5200 C. 5400 D. 5600 E. 5800

12. Suppose the two real roots of the equation 3 8 0 are and . If the arithmetic mean of $\frac{1}{\ }$ and $\frac{1}{\ }$ is 2, what is the value of ?

A. 2 B. 1 C. 4 D. 1 E. 2

13. Two different numbers are picked up from 1, 2, 3, 4, and 5 sequentially to form the tens and unit digits of a two-digit number. The probability that the two-digit number is less than 40 is

A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{3}{5}$ D. $\frac{4}{5}$ E. 1

14. The solution of the equation 6 2 2 6 2 is

A. 1 B. $\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1

15. Peter first faced east and walked for kilometers, and then he turned 150 degrees to the right and walked 3 kilometers. Now he was $\sqrt{3}$ kilometers away from the starting point. Find the value of .

A. $3\sqrt{3}$ B. $\frac{\sqrt{3}}{3}$ C. $\frac{1}{3}$ D. 3 E. $2\sqrt{3}$ or $\sqrt{3}$

1. Given that the graph of the quadratic function ( ) passes through the point (5, 0). Its axis of symmetry is 2, and the minimum value of ( ) is 9.
   (a) Find the values of , and . (3 marks)
   (b) Find the expression of the function after shifting the graph to the left by 3 units, and then shifting it up by 3 units. (2 marks)
   (c) Let ( ) (3sin). Find the maximum and minimum values of ( ). (3 marks)

2. Let { } be a geometric sequence with 1 as the first term, and sequence { } is given as $\frac{}{4}$.
   Suppose that , 4 , 16 form an arithmetic sequence.
   (a) Find the general terms for { } and { } . (3 marks)
   (b) Find , sum of the first terms for { } , and , sum of the first terms for { } .
   (5 marks)

3. Suppose the eccentricity of the ellipse : — — 1 (0) is $\frac{\sqrt{2}}{2}$, and $(2\sqrt{2}, 4)$ is a point lying on
   (a) Find the equation of . (3 marks)
   (b) Let : be one line which does not pass through the origin and is not parallel to the coordinate axes. There are two intersections and for the line and the ellipse . The midpoint of the line segment is , and the slope of the line is . Show that 2. (5 marks)

4. (a) Express $\sqrt{3}$cos sin in the form of cos( ), and find the range of $\sqrt{3}$cos sin .
   (2 marks)
   (b) Solve $\sqrt{3}$cos sin 1 for 0 2 . Answer is to be presented in radians. (3 marks)
   (c) If $\sqrt{3}$cos sin $\frac{1}{2}$, and 0 , find sin $\frac{}{2}$ $\frac{}{12}$. (3 marks)

5. Prove by mathematical induction that 3 5 is divisible by 14 for any positive integer .
   (8 marks)

1	B
2	C
3	E
4	B
5	D
6	A
7	A
8	D
9	D
10	A
11	B
12	E
13	C
14	C
15	E

(Answers for Part II start from next page)

1. (a) The coefficients of the quadratic function satisfy the following system

$$25 \quad 5 \quad 0$$
$$4 \quad .$$
$$4 \quad 2 \quad 9$$

Solving the equation system yields

$$1, \quad 4, \quad 5.$$

The function is

$$(\ ) \quad 4 \quad 5.$$

(b) $(\quad 1) \quad 6 \quad 2 \quad 5.$

(c) We have $(\ ) \quad (3\sin \ ) \quad 4(3\sin \ ) \quad 5 \quad (3\sin \quad 2) \quad 9.$

Since $3 \quad 3\sin \quad 3 \quad 2 \quad [\ 3,3]$ the function obtains its maximum value at $3\sin \quad 3$, that is,

$$\text{Max} \ (\ ) \quad (\ 3) \quad 16.$$

The function attains its minimum value at $3\sin \quad 2$, that is,

$$\text{Min} \ (\ ) \quad (2) \quad 9.$$

2. (a) Let . As $,4 \ ,16$ are in arithmetic progression, so $16 \quad 8$, i.e.,

$$16 \quad 8 \quad .$$

Since 1, we have $\frac{1}{4}$. Thus

$$\frac{1}{4} \quad , \quad \frac{}{4}.$$

(b) From the geometric sum formula, we get $\frac{4}{3} 1 \ \frac{1}{4} \ .$

Since

$$\frac{1}{4} \ \frac{2}{4} \ \frac{3}{4} \ \frac{4}{4} \ \cdots \ \frac{1}{4} \ \frac{}{4}, \qquad (1)$$

we have

$$4 \quad 1 \ \frac{2}{4} \ \frac{3}{4} \ \frac{4}{4} \ \cdots \ \frac{}{4}. \qquad (2)$$

Subtracting (1) from (2) gives

$$3 \quad 1 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \cdots \ \frac{1}{4} \ \frac{}{4} \ \frac{4}{3} 1 \ \frac{1}{4} \ \frac{}{4}.$$

Hence

$$\frac{4}{9} 1 \ \frac{1}{4} \ \frac{}{3\ 4}.$$

3. (a) and satisfy the system

$$\frac{\sqrt{\quad}}{} \quad \frac{\sqrt{2}}{2}.$$
$$\frac{8}{} \quad \frac{16}{} \quad 1$$

This implies that 32, 16. Thus the equation is

$$\frac{}{16} \quad \frac{}{32} \quad 1.$$

(b) Combining the equations of the line and the ellipse $\frac{}{16} \quad \frac{}{32} \quad 1$, we get

$$\frac{}{16} \quad \frac{(\quad)}{32} \quad 1.$$

That is, (2) 2 32 0. Then the coordinates of are

$$\frac{}{2}, \quad \frac{2}{2}.$$

Therefore, the slope of the line segment OM is

$$\frac{2}{}.$$

Consequently, 2

4. (a) $\sqrt{3}\cos \quad \sin \quad 2(\frac{\sqrt{3}}{2}\cos \quad \frac{1}{2}\sin \quad) \quad 2(\cos\frac{}{6}\cos \quad \sin\frac{}{6}\sin \quad) \quad 2\cos(\quad \frac{}{6})$.

It follows from the above that 2 $\sqrt{3}\cos \quad \sin \quad 2$.

(b) From the result of (a), we get $\sqrt{3}\cos \quad \sin \quad 2\cos(\quad \frac{}{6})$.

Since $\sqrt{3}\cos \quad \sin \quad 1$, so $\cos(\quad \frac{}{6}) \quad \frac{1}{2}$.

Since 0 2 , we have $\frac{}{6} \quad \frac{2}{3}$ or $\frac{4}{3}$, that is $\frac{}{2}$ or $\frac{7}{6}$.

(c) Since $\sqrt{3}\cos \quad \sin \quad \frac{1}{2}$, therefore $2\cos(\quad \frac{}{6}) \quad \frac{1}{2}$, that is $\cos(\quad \frac{}{6}) \quad \frac{1}{4}$.

By the double-angle formula, we have $1 \quad 2\sin(\frac{}{2} \quad \frac{}{12}) \quad \frac{1}{4}$, so $\sin(\frac{}{2} \quad \frac{}{12}) \quad \frac{5}{8}$. And since

0 , we get $0 \quad \frac{}{2} \quad \frac{}{12}$.

Therefore

$$\sin(\frac{}{2} \quad \frac{}{12}) \quad \sqrt{\frac{5}{8}} \quad \frac{\sqrt{10}}{4}.$$

5. Proof: Let ( ) be the statement "3 5 is divisible by 14".

(1). When 1 ,

$$3 \quad +5 \quad =3 \ +5 \ =729+125=854 ,\ 854 \quad 14 \quad 16 .$$

Therefore, (1) is true.

(2). Assume that ( ) is true for (ℤ) , i.e., 3 5 is divisible by 14, where ℤ .

So when 1(ℤ) ,

$$3 \qquad 5$$
$$3 \qquad 5$$
$$3 \ \ 3 \qquad 5 \ \ 5$$
$$81\ 3 \qquad 25\ 5$$
$$56\ 3 \qquad 25\ 3 \qquad 25\ 5$$
$$=56\ 3 \qquad 25 \ \ 3 \qquad 5 \qquad ,$$

Where 56 3 is divisible by 14, and according to the assumption, 25 (3 5) is also divisible by 14.

In other words, (1) is also true.

By (1), (2) and the Principle of Mathematical Induction, the statement is true for all positive integers .